1.2 Worksheet: The Concept of Limits

Purpose: One of the best ways to learn math is practice. Often we can't tell what we don't understand until we try to do it. The goal of this worksheet is to give you lots of problems to try. If you get stuck or make mistakes the important thing to do is maintain a growth oriented mindset! This is just practice, and there will be more practice before you are done. Take your time, remember our goals for productive group work, and ask questions!

1. **Estimating limits using a table of values:**

   (a) Fill in the table to estimate the left and right limits $\lim_{x\to 0^-} \frac{\tan x}{x}$ and $\lim_{x\to 0^+} \frac{\tan x}{x}$.

x	$\frac{\tan x}{x}$
-0.1	
-0.01	
-0.001	

x	$\frac{\tan x}{x}$
0.1	
0.01	
0.001	

   What do you conclude from this data?

   (b) Fill in the table to estimate the left and right limits $\lim_{x\to 2^-} \frac{x-2}{|x-2|}$ and $\lim_{x\to 2^+} \frac{x-2}{|x-2|}$.

x	$\frac{x-2}{\lvert x-2\rvert}$
-0.1	
-0.01	
-0.001	

x	$\frac{x-2}{\lvert x-2\rvert}$
0.1	
0.01	
0.001	

   What do you conclude from this data?

2. **Some limits don't exist!:**

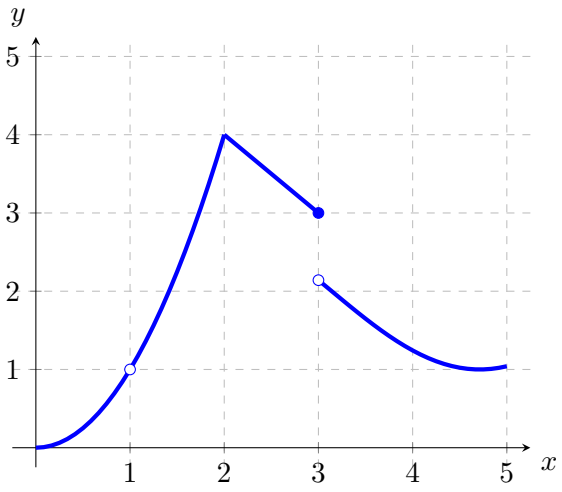
   a) Sketch a graph of the function $f(x) = \frac{1}{x}$. Make a table of values for the left and right limits as $x \to 0$ for $f(x) = \frac{1}{x}$.

   b) Sketch a graph of the function $h(x) = \ln(x)$. Make a table of values for the right limit $\lim_{x\to 0^+} \ln(x)$.

3. **Using algebra to find a limit:** Compute the following limit exactly by canceling out the factor of $x + 2$ from the numerator and denominator.

$$\lim_{x\to -2} \frac{x+2}{x^2+2x}$$

4. **Using a graph to find limits:** Use the graph to compute the limits.



a) $\lim_{x\to 1^-} f(x)$ c) $\lim_{x\to 1} f(x)$ e) $\lim_{x\to 3^+} f(x)$ g) $\lim_{x\to 2} f(x)$

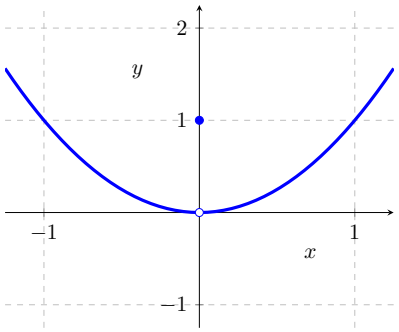
b) $\lim_{x\to 1^+} f(x)$ d) $\lim_{x\to 3^-} f(x)$ f) $\lim_{x\to 3} f(x)$

5. **Using all of our tools and being careful:** For each of the following limits decide if the left limit, the right limit, and the limit exist or not. Feel free to use tables or graphs to help.

   Helpful reminder: We say that a one sided limit to a exists and is equal to L if when we make x arbitrarily close to a the value of $f(x)$ becomes arbitrarily close to L. We say that the limit as x approach a exists if both the left and right limits exist and agree.

   (a) Let

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}.$$



   Does $\lim_{x\to 0} f(x)$ exist?

   (b) Does $\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right)$ exist?